APÉNDICE XIV.- COMMENTS ON KOLB & TURNER'S "THERMODYNAMICS IN THE EXPANDING UNIVERSE," (nota [1845])

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Everything in Kolb & Turner's chapter V is correct up to equation (5.7, p.116). This is the Boltzmann equation for a Robertson-Walker metric. From there on all steps are flawed. Let ψ be the "test" particle. If it is to collide with a set of different particles, say a, b, c, d, ... then *the Boltzmann equation admits only binary collisions*:

$$\psi + a \leftrightarrow \psi' + a' \qquad (1)$$
$$\psi + b \leftrightarrow \psi' + b'$$

and so on. The primes denote the same particles with different energies and momenta after the collision. The $\leftrightarrow$ symbol indicates that, if charge parity invariance holds true (microscopic reversibility), the inverse collisions exist and therefore:

$$\begin{aligned} C(f) &\approx \sigma(\psi, a \leftrightarrow \psi', a')(f(\psi)f(a) - f(\psi')\psi(a')) + \\ &+ \sigma(\psi, b \leftrightarrow \psi', b')(f(\psi)f(b) - f(\psi')\psi(b')) + \\ &+ \ldots\ldots. \end{aligned} \qquad (2)$$

Where the sigmas (σ's) (mus μ's in the authors' language) are the collision cross sections for each binary collision specified in (1). Therefore the symbolic expression for many particles collisions

$$\psi + a + b + \ldots. \leftrightarrow i + j + \qquad (3)$$

is meaningless, even if the particle set $(i, j, \ldots.)$ is not different from the set $(\psi, a, b, \ldots.)$, as appears to be the case. Whence, *equation* (5.8, ps.116-117) *is not only meaningless, it is wrong. Multiple collisions (more than 3) are outside the domain of Boltzmann's model.*

The substitution of the f's by their quantum statistical counterparts is inconsistent with the classical expression for the drift term in (5.7, p. 116). They should never have been introduced, so the so called "second simplification" (p.117)[1846] is uncalled for.

The statement that "$f_i(E_i) = \exp[-(E_i - \mu_i)/T]$ *for all species in kinetic equilibrium,*"[1847] is suspect. As clearly shown in Bernstein's book (ref. 2 end of chapter), "*there is no equilibrium*

[1845] Edward Kolb & Michael Turner, *The Early Universe* (1994): chapter v

[1846] Edward Kolb & Michael Turner, *The Early Universe* (1994): 117-118: "*The second simplification is the use of Maxwell-Boltzmann statistics for all species [of particles] instead of Fermi-Dirac for fermions and Bose-Einstein for bosons. In the absence of Bose condensation or of Fermi degeneracy, the blocking and stimulated emission factors can be ignored,* $1 \pm f \cong 1$, *and* $f_i(E_i) = \exp[-(E_i - \mu_i)/T]$ *for all species in kinetic equilibrium. With these two assumptions the Boltzmann equatioin may be cast in the familiar form*

$$\dot{n}_\psi + 3H n_\psi = -\int \begin{matrix} d\Pi_\psi\, d\Pi_a d\Pi_b \ldots\ldots d\Pi_i d\Pi_j \ldots (2\pi)^4 |M|^2 \\ \times \delta^4(p_i + p_j \ldots - p_\psi - p_a - p_b \ldots)[f_a f_b \ldots f_\psi - f_i f_j \ldots] \end{matrix} \qquad (5.11),$$

where as usual, $H = \dot{R}/R$. *The significance of the individual terms is manifest: The* $3Hn\psi$ *term accounts for the dilution effect of the expansion of the Universe, and the right hand side of (5.11) accounts for interactions that change the number of* ψ *'s present.*"

[1847] Edward Kolb & Michael Turner, *The Early Universe* (1994):118

distribution for a Robertson-Walker gas,"[1848] so that the validity of such results is questionable. Equation (5.11) is once more wrong. The integral in the right hand side should contain only sum of products of binary collisions $f(\psi)\, f(n)$, where $n = i, a, b, ...$ whatever.

Worst of all is the statement about *"accounting for ... the expansion of the Universe*" (p. 118, see note 1). The authors introduce "*the entropy density* s*"* (p. 118).[1849] What entropy? According to Boltzmann, such quantity, for the i^{th} species, is given by

$$s_i \approx \int dp_i f_i \ln(f_i) \qquad (4)$$

provided f_i is the exact solution to the here incorrectly derived Boltzmann equation. If $f_i = f_i^{(0)}$, the Maxwell-Boltzmann equation for the i^{th} species in equilibrium turns out to be the ordinary entropy density for an ideal gas. But the ideal gas is one thing, the Robertson-Walker gas, for which $f_i^{(0)}$ does not exist in equilibrium, is something else. So what is the meaning of s_i, its conservation and thus the physical meaning of (5.13)? It seems to me that all of these steps are in opposition to Boltzmann's philosophy.[1850] This implies further that equation (5.16) is wrong[1851] and the authors' pretence that armed with their Boltzmann technology we can face the "*non-equilibrium thermodynamics of the Universe*" is meaningless.[1852] The whole chapter based on these rather superfluous, undefined and vague concepts is out of place. Needless to enter into more details, it is just a waste of time.

1848 Jeremy Bernstein, *Kinetic Energy in the Expanding Universe* (1988): 23

1849 Edward Kolb & Michael Turner, *The Early Universe* (1994):118: "*Using the conservation of entropy per comoving volume (*sR^3 *is constant), it follows that* $\dot{n}_\psi + 3H\, n_\psi = s\dot{Y}$ *(*5.13)."

1850 Ludwig Boltzmann, *Theoretical Physics and Philosophical Problems* (1974): 202:"*Every hypothesis must derive indubitable results from mechanically well defined assumptions by mathematically correct methods.*" The problem is that Kolb & Turner do not part from mechanically well defined assumptions, nor do they use mathematically correct methods.

1851 Edward Kolb & Michael Turner, *The Early Universe* (1994):119: "*the Boltzmann equation can be rewritten as*

$$\frac{dY}{dx} = -\frac{x}{H(m)s}\int \begin{matrix} d\Pi_\psi\, d\Pi_a d\Pi_b ......d\Pi_i d\Pi_j ...(2\pi)^4 |M|^2 \\ \times \delta^4(p_i + p_j ... - p_\psi - p_a - p_b ...)[f_a f_b ... f_\psi - f_i f_j ...] \end{matrix} \qquad (5.16)$$

1852 Edward Kolb & Michael Turner, *The Early Universe* (1994):119: "*We will now consider some specific applications of the [Boltzmann] formalism we have developed to treat non-equilibrium thermodynamics [in the expanding Universe].*"